

Application of Grey System Theory in Evaluating Innovative Environment for an Industrial Cluster

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Abstract Evaluation on an industrial cluster is the key link from theory to practice. So based on the in-depth analysis of the innovative environment for an industrial cluster, an evaluation index system was set up for the innovative environment on which most of the influencing factors are characterized with grey, fuzzy and hard to quantify in terms of technical economy. Therefore, the analytical hierarchy process (AHP) and the theory of grey system is adaptable to those systems which are hard to quantify accurately for developing models were introduced in to set up a comprehensive evaluation model for the grey hierarchy in an innovative environment for an industrial cluster. Then the relevant evaluation indices were obtained and an example is given to verify the result of evaluation.

Key words Grey system theory; Industrial cluster; Innovative environment; Index system

1 Introduction

Tracey Paul^[1], Martin, R. Sunley^[2] and some others have discovered that in the innovative regions, the innovative manufacturers clustered in some special zones with their respectively independent operation may wield the favorable environment characteristics in this zone. Ahuja Gautam^[3] believes that the innovative environment is the life of the industrial cluster compared with those in the isolated places, which can enable enterprises within the industrial cluster to innovate fast with less speed and lower costs. At the same time, those manufacturers are pressed into differing from others in an innovative way under the great pressure from peer competition, consistent comparison and similar basic environment (such as labor and equipment costs), so that the pressure for innovation will be increased gradually, resulting in the faster development of enterprises than those in other sites, even though an individual is hard to remain top for a long time.

2 Enterprise's Innovative Environment

The innovative environment for an enterprise is not a regional concept (though it is affected by the region, sometimes its effects are rather substantial), but an inter-regionally functional concept. The innovative environment for an industrial cluster with regional characteristics emphasizes society-rooted qualities, competitive and cooperative relationships and division-of-labor network characteristics. To make it more specific, the differences between the innovative environment for an industrial cluster and that for an individual enterprise are listed in Table 1.

Table 1 A Comparison Between an Industrial Cluster and an Enterprise's Innovative Environment

Project	An industrial cluster	An enterprise
Range	Regional	Inter-regional
Credit rating	High	Low
Division-of-labor and cooperative degree	High	Low
Competition	Strong	Weak
Capital and resources	Properly capital exclusiveness	Properly capital exclusiveness
Trade costs	Low	High
Implied knowledge broadcasting	Appropriate	Hard
Motive force on innovation	Strong	Average
Society-rooted qualities	Strong	Weak

3 Means of Evaluation on Innovative Environment for an Industrial Cluster

3.1 An evaluation index system of the innovative environment for an industrial cluster

Factors influencing the innovative environment for an industrial cluster are so many and complex that the evaluation system must be set up from multiple angles and aspects so as to reflect the innovative environment accurately. Therefore, the evaluation system of the innovative environment for an industrial cluster is shown in Table 2 by drawing on the foreign and domestic research fruits in this regard.

Table 2 Evaluation Index System of Innovative Environment for an Industrial Cluster

Evaluation objective	The innovative environment for an industrial cluster U				
The sub-layer evaluation factor U_i	1 st	The factors of innovative service environment U_1	The factors of innovative technology environment U_2	The factors of innovative human environment U_3	The factors of innovative market environment U_4
The sub-layer evaluation factor U_{ij}	2 nd	Supporting power on innovation from economic policies U_{11}	The quantity of universities and science and research institutes U_{21}	The scientific and cultural level of human resources U_{31}	The increasing rate of sales amount U_{41}
		Supporting power on innovation from laws and local regulations U_{12}	R&D expenditure / GDP U_{22}	The motive force of consumers' concept of innovation U_{32}	The intermediary organizations' function in innovation U_{42}
		The reasonable degree of legal environment U_{13}	The constructive level of regional public database U_{23}	The mobility of trained people U_{33}	The increasing rate of investment amount U_{43}
		The perfected degree of legal environment U_{14}	Technological cooperation degree among enterprises U_{24}	The average occupation number of books U_{34}	Technology-intensified industry output value / GDP U_{44}
		The perfected degree of industrially infrastructural facilities U_{15}	The co-operational and communicational degree between enterprises and universities and science and research institutes U_{25}	The number of people engaged in R&D per 10,000 employees U_{35}	Credit rating U_{45}
		The perfected degree of living infrastructural facilities U_{16}	The proportion of microelectronic control facilities in production equipment U_{26}	Teamwork spirits U_{36}	The reasonable degree of market structure U_{46}
		The protected power of intellectual property right U_{17}	The degree of opening to the outside world U_{27}	The attraction of better living conditions U_{37}	The per capita GDP U_{47}
		The transparency of governmental decision-making U_{18}	The power to assimilate the advanced technology U_{28}	Entrepreneurship	The intensity of market competition U_{48}
		The protected power of secured system U_{19}	The proportion of the enterprises with internal web works in all enterprises (large or middle-sized) U_{29}	Proportion of talent flow	Industrial growth rate
		Local traditional culture	Conversion rate of scientific and technological achievements (%)	The proportion of R&D personnel (%)	Brand recognition
The power of smooth financing U_{110}	The proportion of R&D investment (%)				

3.2 Comprehensive evaluation model of grey hierarchy in innovative environment for an industrial cluster

Grey system theory^[4-6] specializes in researching insufficient data and indefiniteness. Its mathematical means is not a statistical one. This makes it more practical when the systematic data and conditions are too few to meet the statistical requirements. In addition, many factors influencing the innovative environment for an industrial cluster are often grey, fuzzy and hard to quantify. Thus this evaluation can not exclude deviations caused by man-made elements such as the evaluators' knowledge level, recognition ability and personal preference. In this sense, the grey system theory can be used to set up a comprehensive evaluation model in an innovative environment for an industrial cluster.

3.2.1 To determine the weight of evaluation index U_i, U_{ij}

When the AHP (Analytic Hierarchy Process)^[7] is used to determine the weight vectors of evaluation index, that is the weight vectors $A = (a_1, a_2, a_3, a_4)$ of evaluation index are $U_i (i = 1, \dots, 4)$, the weight vectors of evaluation index $U_{1j} (j = 1, \dots, 11)$, $U_{2j} (j = 1, \dots, 11)$, $U_{3j} (j = 1, \dots, 10)$, $U_{4j} (j = 1, \dots, 10)$ are respectively $A_1 = (a_{11}, \dots, a_{111})$, $A_2 = (a_{21}, \dots, a_{211})$, $A_3 = (a_{31}, \dots, a_{311})$, $A_4 = (a_{41}, \dots, a_{411})$.

3.2.2 To determine the standards of evaluation

First, the quantitative indices in the second rank evaluation U_{ij} should be categorized dimensionless, and then be graded on the conversion rate of 9 points. Accordingly, the quantitative indices can be categorized into A, B, C, D, E, and the corresponding points are respectively 9, 7, 5, 3, 1. The indices are between two neighboring ranks, grading subsequently 8, 6, 4, and 2.

3.2.3 To grade by experts and determine evaluation sample matrix

P evaluation experts are organized to grade the innovative environment for the industrial cluster according to the above evaluation category and the evaluation sample matrix of the graded innovative environment for the industrial cluster is as following:

$$D = \begin{bmatrix} d_{11} & d_{12} & \dots & d_{1p} \\ d_{21} & d_{22} & \dots & d_{2p} \\ \dots & \dots & \dots & \dots \\ d_{421} & d_{422} & \dots & d_{42p} \end{bmatrix}$$

3.2.4 Evaluation on the grey system and its calculation

First, the grey system is categorized into five ranks: A, B, C, D, E and its corresponding grey system ranks and bleaching weight functions^[12] are as following:

- Grey system rank A $e = 1$, grey value $\otimes_1 \in [0, 9, \infty]$, bleaching weight function f_1 ;
- Grey system rank B $e = 2$, grey value $\otimes_1 \in [0, 7, 14]$, bleaching weight function f_2 ;
- Grey system rank C $e = 3$, grey value $\otimes_1 \in [0, 5, 10]$, bleaching weight function f_3 ;
- Grey system rank D $e = 4$, grey value $\otimes_1 \in [0, 3, 6]$, bleaching weight function f_4 ;
- Grey system rank E $e = 5$, grey value $\otimes_1 \in [0, 1, 2]$, bleaching weight function f_5 .

Then the grey system evaluation coefficients are calculated into

$$x_{ije} (e = 1, \dots, 5) : x_{ije} = \sum_{k=1}^p f_e(d_{ijk}). \text{ For the index } U_{ij}, \text{ the affiliated grey system evaluation gross}$$

$$\text{value is } x_{ij} = \sum_{e=1}^5 x_{ije}.$$

Finally, the grey evaluation weight $r_{ije} = \frac{x_{ije}}{x_{ij}}$ of the e^{th} rank in the evaluation grey system can be got by p experts for evaluation index U_{ij} , and the grey system weight vectors are $r_{ij} = (r_{ij1}, r_{ij2}, r_{ij3}, r_{ij4}, r_{ij5})$. In this sense, U_{ij} affiliated indices grey system matrix R can be got in every evaluation grey system ranks.

$$R = \begin{bmatrix} r_{11} & r_{12} & \dots & r_{15} \\ r_{21} & r_{22} & \dots & r_{25} \\ \dots & \dots & \dots & \dots \\ r_{421} & r_{422} & \dots & r_{425} \end{bmatrix}$$

3.2.5 To make a comprehensive evaluation on U and U_i

A comprehensive evaluation on U_i should be made firstly, and the gross grey evaluation matrix R can be got from its evaluation result $B_i = A_i \cdot R_i = (b_{i1}, b_{i2}, b_{i3}, b_{i4}, b_{i5})$. After that, a comprehensive evaluation should be made on U , that is the innovative environment for an industrial cluster, and its result is $B = A \cdot R = (b_1, b_2, b_3, b_4, b_5)$. At last, the grey system ranks for the industrial cluster can be made according to maximum principle. But sometimes, this principle will lose its function for it causes too much lost in information. At this time, it is necessary to further process B to calculate

its single value. Each grey system rank will be given values according to the grey level, and the evaluation vectors in the grey system ranks are $C = (d_1, d_2, d_3, d_4, d_5) = (9, 7, 5, 3, 1)$. Then the comprehensive evaluation value Z can be calculated by formula $Z = B \cdot C^T$. Compared with grey system ranks, the innovative environment for the industrial cluster can be evaluated by Z .

4 Exemplification

Based on the above evaluation index system and evaluation model, the comprehensive evaluation on the innovative environment for an industrial cluster can be made specifically as following:

4.1 To determine the weight of evaluation index

The weight vectors of U_i and U_{ij} can be determined by AHP:

$$D_1 = 0.32; D_2 = 0.52; D_3 = 0.09; D_4 = 0.07; D_5 = 0.42; D_6 = 0.36; D_7 = 0.11; D_8 = 0.11; D_9 = 0.5; \\ D_{10} = 0.25; D_{11} = 0.25; D_{12} = 0.32; D_{13} = 0.56; D_{14} = 0.12; D_{15} = 0.5; D_{16} = 0.5; D_{17} = 0.40; D_{18} = 0.18; \\ D_{19} = 0.11; D_{20} = 0.31; D_{21} = 0.22; D_{22} = 0.27; D_{23} = 0.09; D_{24} = 0.10; D_{25} = 0.09; D_{26} = 0.23; \\ D_{27} = 0.12; D_{28} = 0.16; D_{29} = 0.29; D_{30} = 0.43; D_{31} = 0.29; D_{32} = 0.13; D_{33} = 0.09; D_{34} = 0.49; \\ D_{35} = 0.37; D_{36} = 0.11; D_{37} = 0.12; D_{38} = 0.40; D_{39} = 0.40; D_{40} = 0.14; D_{41} = 0.14; D_{42} = 0.32$$

4.2 To grade the innovative environment for an industrial cluster

Five specialists are organized to grade the innovative environment for the industrial cluster according to the established evaluation standard. The evaluation sample matrix can be made as following:

$$D = \begin{bmatrix} 2.0 & 1.5 & 2.2 & 1.0 & 1.5 \\ 1.5 & 2.0 & 1.8 & 1.5 & 2.0 \\ 1.5 & 1.5 & 1.0 & 1.2 & 1.7 \\ 2.0 & 1.8 & 1.7 & 1.5 & 1.2 \\ 7.5 & 7.0 & 8.0 & 7.7 & 7.5 \\ 5.5 & 6.0 & 4.9 & 6.7 & 5.0 \\ 8.0 & 8.5 & 8.2 & 8.5 & 8.0 \\ 1.5 & 1.2 & 1.2 & 1.1 & 1.0 \\ 5.5 & 4.7 & 6.5 & 4.5 & 6.8 \\ 5.0 & 5.1 & 5.5 & 4.0 & 4.5 \\ 4.9 & 4.8 & 5.5 & 5.9 & 4.5 \\ 5.8 & 5.5 & 4.5 & 5.0 & 5.0 \\ 6.0 & 6.0 & 5.0 & 4.5 & 4.9 \\ 4.0 & 3.5 & 3.0 & 2.5 & 2.5 \\ 3.0 & 3.5 & 2.9 & 2.5 & 3.0 \\ 5.5 & 4.5 & 6.0 & 5.8 & 4.9 \\ 7.5 & 7.2 & 7.8 & 7.0 & 7.8 \\ 5.0 & 4.5 & 4.7 & 5.8 & 5.0 \\ 3.0 & 3.5 & 4.0 & 2.9 & 2.5 \\ 3.0 & 2.9 & 3.1 & 2.8 & 3.0 \\ 2.0 & 2.5 & 3.0 & 2.5 & 2.0 \\ 6.0 & 6.5 & 7.5 & 7.0 & 6.8 \\ 1.0 & 1.2 & 1.5 & 1.5 & 1.0 \\ 3.0 & 6.8 & 6.5 & 7.0 & 4.8 \\ 6.0 & 1.9 & 5.5 & 6.5 & 6.0 \\ 8.0 & 7.0 & 2.5 & 7.5 & 7.0 \\ 7.9 & 7.5 & 8.0 & 5.5 & 8.0 \\ 7.0 & 2.5 & 7.5 & 6.8 & 7.2 \\ 6.9 & 7.2 & 7.0 & 7.1 & 3.8 \\ 4.0 & 4.5 & 3.5 & 2.2 & 4.2 \\ 6.0 & 6.5 & 5.5 & 5.9 & 5.8 \\ 1.0 & 2.5 & 1.5 & 2.2 & 2.5 \\ 5.5 & 6.0 & 5.5 & 6.5 & 5.0 \\ 7.0 & 7.5 & 6.8 & 6.5 & 5.0 \\ 7.0 & 6.9 & 4.5 & 2.5 & 7.0 \\ 5.0 & 4.5 & 5.5 & 1.8 & 4.9 \\ 6.0 & 5.5 & 5.8 & 5.9 & 6.0 \\ 8.5 & 7.5 & 8.5 & 6.0 & 8.0 \\ 6.5 & 5.5 & 6.0 & 4.0 & 6.0 \\ 7.0 & 6.5 & 1.8 & 7.2 & 6.8 \\ 5.5 & 7.5 & 8.0 & 3.7 & 7.2 \\ 4.5 & 1.5 & 4.0 & 4.2 & 4.5 \end{bmatrix}$$

4.3 To make a comprehensive evaluation

The innovative environment for the industrial cluster can be evaluated on the above evaluation model and above data.

4.3.1 To calculate grey evaluation indices and weight vectors

For the evaluation index U_{11} , the e^{th} evaluation coefficients in the evaluation grey system are x_{11e} .

$$\begin{aligned}
 e = 1: & \quad x_{11} = f_1(d_{11}) + f_1(d_{12}) + f_1(d_{13}) + f_1(d_{14}) + f_1(d_{15}) = 0.9111 \\
 e = 2: & \quad x_{12} = f_2(d_{11}) + f_2(d_{12}) + f_2(d_{13}) + f_2(d_{14}) + f_2(d_{15}) = 1.1714 \\
 e = 3: & \quad x_{13} = f_3(d_{11}) + f_3(d_{12}) + f_3(d_{13}) + f_3(d_{14}) + f_3(d_{15}) = 1.6400 \\
 e = 4: & \quad x_{14} = f_4(d_{11}) + f_4(d_{12}) + f_4(d_{13}) + f_4(d_{14}) + f_4(d_{15}) = 2.7333 \\
 e = 5: & \quad x_{15} = f_5(d_{11}) + f_5(d_{12}) + f_5(d_{13}) + f_5(d_{14}) + f_5(d_{15}) = 2.0000
 \end{aligned}$$

For the evaluation index U_{11} , the affiliated gross evaluation value in every grey system is $x_{11} = \sum_{e=1}^5 x_{11e} = 8.4558$.

For evaluation index U_{11} (Supporting power on innovation from economic policies), evaluators claim that the e^{th} grey evaluation weights in evaluation grey system are r_{11e} :

$$\begin{aligned}
 e = 1, & \quad r_{11} = x_{11}/x_1 = 0.9111/8.4558 = 0.1077 \\
 e = 2, & \quad r_{12} = x_{12}/x_1 = 1.1714/8.4558 = 0.1385 \\
 e = 3, & \quad r_{13} = x_{13}/x_1 = 1.6400/8.4558 = 0.1940 \\
 e = 4, & \quad r_{14} = x_{14}/x_1 = 2.7333/8.4558 = 0.3233 \\
 e = 5, & \quad r_{15} = x_{15}/x_1 = 2.0000/8.4558 = 0.2365
 \end{aligned}$$

In this way, when the evaluation index is U_{11} that reflects the supporting power on innovation from economic policies, the grey evaluation weight vectors are r_{11} :

$$r_1 = (r_{11}, r_{12}, r_{13}, r_{14}, r_{15}) = (0.1077, 0.1385, 0.1940, 0.3233, 0.2365)$$

The same is true of calculating the other evaluation grey systems of evaluation indices and gross grey system evaluation coefficients. After that, it may be calculated respectively into r_2, \dots, r_{42} ; compared with grey evaluation matrix R_1, R_2, R_3, R_4 .

$$R = \begin{bmatrix} 0.1077 & 0.1385 & 0.1940 & 0.3233 & 0.2365 \\ 0.1203 & 0.1547 & 0.2165 & 0.3609 & 0.1476 \\ 0.0899 & 0.1155 & 0.1617 & 0.2696 & 0.3633 \\ 0.1104 & 0.1419 & 0.1986 & 0.3311 & 0.2180 \\ 0.3719 & 0.4097 & 0.2184 & 0.0000 & 0.0000 \\ 0.2530 & 0.3252 & 0.3516 & 0.0702 & 0.0000 \\ 0.4380 & 0.3936 & 0.1684 & 0.0000 & 0.0000 \\ 0.0764 & 0.0982 & 0.1376 & 0.2293 & 0.4585 \\ 0.2531 & 0.3254 & 0.3320 & 0.0895 & 0.0000 \\ 0.2114 & 0.2718 & 0.3615 & 0.1553 & 0.0000 \\ 0.2270 & 0.2919 & 0.3640 & 0.1171 & 0.0000 \\ 0.2276 & 0.2927 & 0.3685 & 0.1112 & 0.0000 \\ 0.2368 & 0.3045 & 0.3617 & 0.0969 & 0.0000 \\ 0.1537 & 0.1976 & 0.2767 & 0.3719 & 0.0000 \\ 0.1453 & 0.1868 & 0.2615 & 0.4065 & 0.0000 \\ 0.2412 & 0.3101 & 0.3593 & 0.0894 & 0.0000 \\ 0.3649 & 0.4114 & 0.2237 & 0.0000 & 0.0000 \\ 0.2188 & 0.2813 & 0.3686 & 0.1313 & 0.0000 \\ 0.1534 & 0.1972 & 0.2761 & 0.3733 & 0.0000 \\ 0.1419 & 0.1825 & 0.2555 & 0.4201 & 0.0000 \\ 0.1411 & 0.1815 & 0.2540 & 0.4234 & 0.0000 \\ 0.3215 & 0.4011 & 0.2774 & 0.0000 & 0.0000 \\ 0.0794 & 0.1020 & 0.1428 & 0.2381 & 0.4377 \\ 0.2594 & 0.3335 & 0.2908 & 0.1163 & 0.0000 \\ 0.2603 & 0.3346 & 0.3238 & 0.0723 & 0.0090 \\ 0.3194 & 0.3722 & 0.2336 & 0.0749 & 0.0000 \\ 0.3665 & 0.3844 & 0.2342 & 0.0149 & 0.0000 \\ 0.3046 & 0.3740 & 0.2476 & 0.0737 & 0.0000 \\ 0.2989 & 0.3771 & 0.2623 & 0.0616 & 0.0000 \\ 0.1749 & 0.2249 & 0.3149 & 0.2852 & 0.0000 \\ 0.2780 & 0.3575 & 0.3421 & 0.0225 & 0.0000 \\ 0.1214 & 0.1560 & 0.2185 & 0.3641 & 0.1400 \\ 0.2595 & 0.3336 & 0.3523 & 0.0546 & 0.0000 \\ 0.3047 & 0.3798 & 0.2876 & 0.0279 & 0.0000 \\ 0.2589 & 0.3329 & 0.2690 & 0.1392 & 0.0000 \\ 0.2040 & 0.2623 & 0.3503 & 0.1664 & 0.0169 \\ 0.2278 & 0.2929 & 0.2921 & 0.1872 & 0.0000 \\ 0.3964 & 0.3905 & 0.2131 & 0.0000 & 0.0000 \\ 0.2604 & 0.3349 & 0.3349 & 0.0698 & 0.0000 \\ 0.2749 & 0.3486 & 0.2415 & 0.1182 & 0.0169 \\ 0.3043 & 0.3495 & 0.2661 & 0.0801 & 0.0000 \\ 0.1768 & 0.2272 & 0.3181 & 0.2354 & 0.0425 \end{bmatrix}$$

4.3.2 To make a comprehensive evaluation on U and U_i ($i = 1, 2, 3, 4$)

$B_1 = A_1R_1, B_2 = A_2R_2, B_3 = A_3R_3, B_4 = A_4R_4$. From B_1, B_2, B_3, B_4 , the gross grey evaluation weight matrix of a certain industrial cluster's innovative environment is

$$R = \begin{bmatrix} B_1 \\ B_2 \\ B_3 \\ B_4 \end{bmatrix} = \begin{bmatrix} 0.2293 & 0.2823 & 0.2747 & 0.1800 & 0.0338 \\ 0.2014 & 0.2554 & 0.2984 & 0.2406 & 0.0041 \\ 0.2799 & 0.3554 & 0.2781 & 0.0822 & 0.0043 \\ 0.1902 & 0.2424 & 0.3150 & 0.2483 & 0.0041 \end{bmatrix}$$

Then a comprehensive evaluation result is $B = AR = (0.2470197 \quad 0.29935555 \quad 0.27199175 \quad 0.1417739 \quad 0.0398594)$, that is a certain industrial cluster's innovative environment made on U . Based on them, Z can be calculated with formula $Z = BC^T$,

$$Z = (0.2470197 \quad 0.29935555 \quad 0.27199175 \quad 0.1417739 \quad 0.0398594) (9, 7, 5, 3, 1)^T = 6.143806$$

From above, the innovative environment for this industrial cluster is so common that the government, enterprises and related organizations should take corresponding innovative strategic measures to perfect the innovative system, to improve its innovative environment for an industrial cluster, to revitalize its innovation and at last to increase its overall competitive power.

5 Conclusion

A grey hierarchy comprehensive evaluation model is set up in an innovative environment for an industrial cluster and researches are made into how to apply this model to evaluation. The means offered in this essay is really feasible and practical for experts to evaluate the innovative environment for an industrial cluster. But it is still necessary to point that the choosing of evaluation index factors should be

readjusted accordingly for different industrial clusters and their different developing phrases, and the index weight vectors should be also different. Only in this way, the evaluation could be more direct and better to conform to the objective reality.

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